

Project 3 (001): Simpson's Second Rules

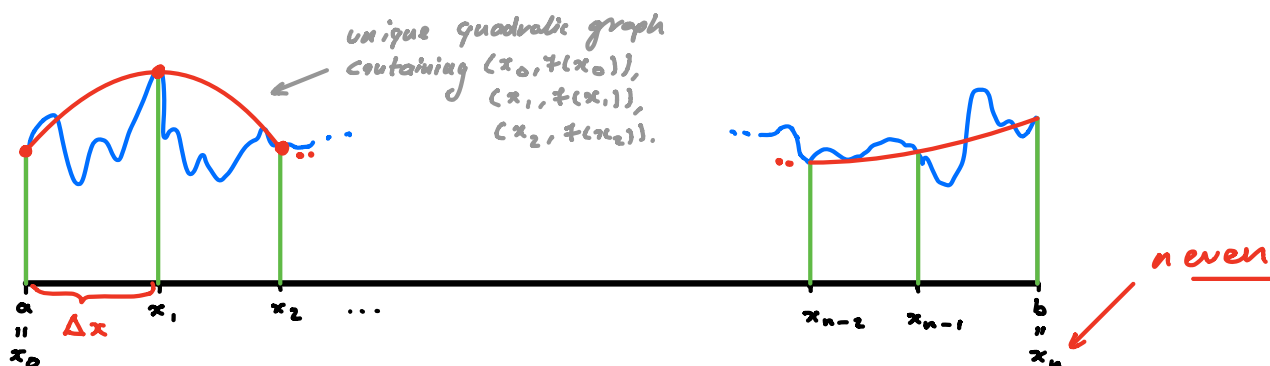
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Student Name and ID : _____

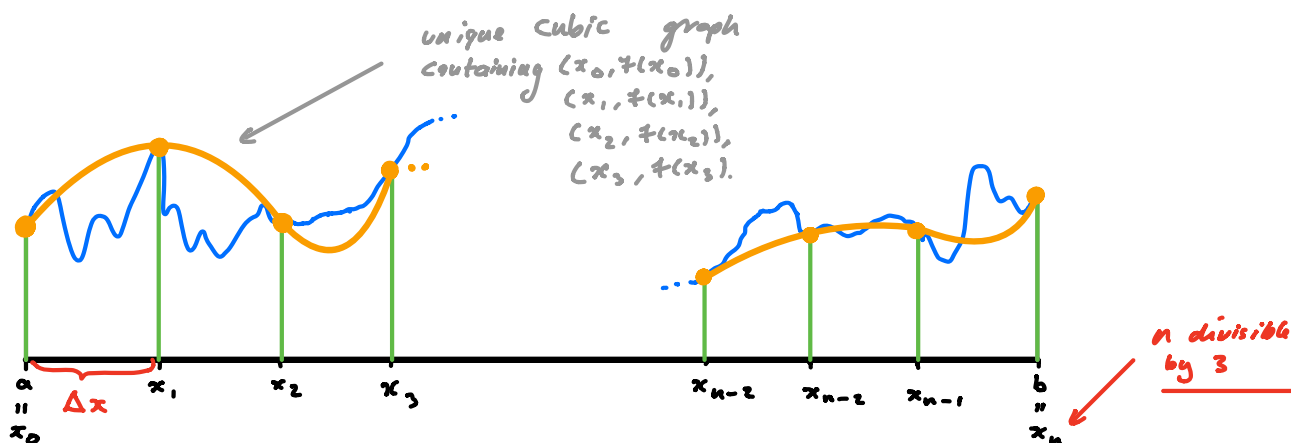
In Simpson's Rule, we approximate a definite integral

$$\int_a^b f(x) dx$$

by making a series of quadratic approximations of the following form:



In this project we will explore a version of Simpson's Rule where we use cubic approximation instead as follows:



$$S_n = \left(\text{sum of area under } 1^{\text{st}} \text{ cubic} \right) + \dots + \left(\text{sum of area under } \left(\frac{n}{3}\right)^{\text{th}} \text{ cubic} \right)$$

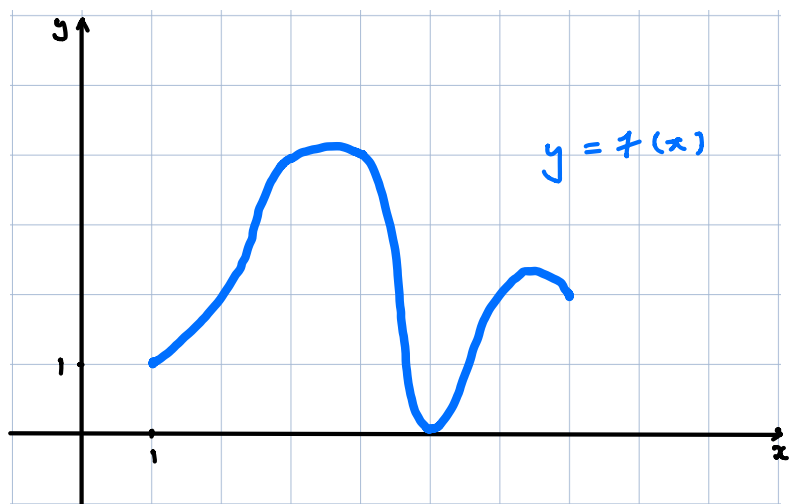
1. Let $f(x) = ax^3 + bx^2 + cx + d$ and $h > 0$. If $x_0 = 0$, $x_1 = h$, $x_2 = 2h$, $x_3 = 3h$ and $\Delta x = h$, verify that

$$\int_{x_0}^{x_3} f(x)dx = \frac{3\Delta x}{8}(f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)).$$

Hint: Calculate each side of the equation separately and compare at the end.

Solution:

2. Using the pattern from question 2, calculate $\mathbb{S}_6$ on the interval $[1,7]$ for the following function:



Hint: There will be two cubic approximations in this case.

Solution:

3. The error bound for Simpson's second rule is given by

$$|E_{\mathbb{S}_n}| \leq \frac{K(b-a)^5}{80n^4},$$

where $|f^{(4)}(x)| \leq K$ for all x in $[a, b]$. (You do not need to justify this).

If $f(x) = \sin(x) - x^5$, $a = 0$ and $b = 1$ give an example of an n such that

$$|E_{\mathbb{S}_n}| < 0.001.$$

Solution: